Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC., 1st Semester Internal Assessment 2023

Full Marks: 10 **Time: 1/2 Hour**

Paper: C2T ALGEBRA

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*Answer **"any one"** of the following questions:*

1.a	Find the roots of $z^n = (1+z)^n$, where n is a positive integer greater than 1. Show that the points which represents them in the z-plane are collinear.	3
1.b	Discuss the reality of the roots of the equation $x^4 + 4x^3 - 12x^2 - 32x + k = 0$ for different real values of k.	4
1.c	Find the equation whose roots are cubes of the roots of cubic $x^3 + 3x^2 + 2 = 0$.	3
2.a	If the equation whose roots are squares of the roots of the cubic $x^3 - ax^2 + bx - 1 = 0$ is identical with this cube, prove that either $a = b = 0, or\ a = b = 3, or\ a\ and\ b$ are the roots of the equation $t^2 + t + 2 = 0$.	4
2.b	State and prove De Moivre's theorem for complex numbers	4
2.c	If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\beta + \gamma - 2\alpha, \gamma + \alpha - 2\beta, \alpha + \beta - 2\gamma$.	2

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*Answer "**any one**" of the following questions:*

1.1	Find the roots of $z^n = (1+z)^n$, where n is a positive integer greater than 1. Show that the points which represents them in the z-plane are collinear.	3
1.2	Discuss the reality of the roots of the equation $x^4 + 4x^3 - 12x^2 - 32x + k = 0$ for different real values of k.	4
1.3	Find the maximum value of $xyz(d - ax - by - cz)$, where all the factors and a, b, c, d are positive.	3
2.1	If a, b, c, d are four positive real numbers and $a + b + c + d = s$, then show that $81abcd \leq (s-a)(s-b)(s-c)(s-d) \leq \frac{81}{256}s^4$.	4
2.2	State and prove De Moivre's theorem for complex numbers	4
2.3	If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $\beta + \gamma - 2\alpha, \gamma + \alpha - 2\beta, \alpha + \beta - 2\gamma$.	2

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Mugberia Gangadhar Mahavidyalaya
Department of Mathematics
Differential Equations
Mathematics (Hons.): 2nd Year: Full Marks 50

Any five from Group -A: $2 \times 5 = 10$

1. Find the general solution of the PDE $uu_x + yu_y = x$.

2. Find the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$.

3. Find the order and degree of the PDE $p \tan y + q \tan x = \sec^2 z$.

4. The PDE $(2x + 3y)p + 4xq - 8pq = x + y$ is
(a) linear (b) non-linear (c) quasi-linear (d) semi-linear.

5. Using Laplace transform, show that $\int\limits_0^\infty te^{-3t} \sin t dt = \frac{3}{50}$.

6. Show that $L\{e^{-2t}(3\cos 6t - 5\sin 6t)\} = \frac{3s-24}{s^2+4s+40}$.

7. Discuss the ordinary and singular point of the differential equation $2x^2\frac{d^2y}{dx^2} + 7x(x + 1)\frac{dy}{dx} - 3y = 0$.

8. Find the solution of the integral equation $y(x) = x + \int\limits_0^x \sin(x-t)y(t)dt, .$

Any eight from Group -B: $5 \times 8 = 40$

1. Solve the SDE $(D+2)x + (D-1)y = 3(t^2 - e^{-t}),\ (2D-1)x - (D+1)y = 3(2t - e^{-t})$.

2. Use Laplace transform to solve $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x$ given that $y(0) = 1,\ \frac{dy(0)}{dx} = 0$.

3. Solve the SDE $(D^2 - 4D + 4)x - y = 0,\ (D^2 + 4D + 4)y - 25x = 16e^t$.

4. Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0,\ z = 1$.

5. Find the equation of the integral surface of $x^2p + y^2q + z^2 = 0$ which passes through the hyperbola $xy = x + y,\ z = 1$

6. Solve in series the equation $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = 0$

7. Solve $y'' + x^2 y = 2 + x + x^2$ about $x = 0$.

8. Prove that $L\Bigg\{f(t-a)H(t-a)\Bigg\} = e^{-as}F(s), s > a > 0$.

9. If $f * g = \int_0^u f(u-t)g(t)dt$, then show that $L\Bigg\{f * g\Bigg\} = F(s) \cdot G(s)$.

10. Solve $(D^2 - 4D + 4)x - y = 0,\ (D^2 + 4D + 4)y - 25x = 16e^t$.

11. Find two families of surfaces that generate the characteristics of $(3y-2u)p+(u-3x)q = 2x - y$.

12. Find the general solution of $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z^2-x^2-y^2}$.

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics

Differential Equations

Mathematics (Hons.): Sem-I(2019): Full Marks 31

Any seven from Group -A: $3 \times 7 = 21$

1. Show that the differential equation of all parabolas with foci at the origin and axis along x-axis is given by
   $y(\frac{dy}{dx})^2 + 2x\frac{dy}{dx} - y = 0$ [V.U.2002]

2. Solve : $(xy \sin xy + \cos xy)ydx + (xy \sin xy - \cos xy)xdy = 0$ [V.U.2002]

3. Reduced the differential equation $x^2p^2 + py(2x + y) + y^2 = 0$ to Clairaut's form by the substitutions $y = u,\ xy = v$, solve it for singular solution and extraneous loci, if any.

4. Show that the substitution $x = e^u$ transforms the equation
   $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \cos x$ into $\frac{d^2y}{du^2} + 3\frac{dy}{du} + 2y = \cos x$. JAM(MA)–2010

5. Prove that the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$ is
   $(x^2 - 2xy - y^2 + 1)dx + (x^2 + 2xy - y^2 - 1)dy = 0$ **V.U(Hons.)-2017**

6. Explain the terms: general solution, a particular solution, a singular solution as applied to an ordinary differential equation.

7. The equation
   $$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, \qquad \textbf{WBSSC 2001} \qquad (1)$$
   (where a and b are fixed constants and λ is an arbitrary parameter which can assume all real values) represents a family of confocal conics. To obtain the differential equation of this family.

8. If $\frac{1}{M-N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x + y)$, then the differential equation $Mdx + Ndy = 0$ has an integrating factor of the form $e^{-\int f(x+y)d(x+y)}$.

9. Show that the general solution of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ can be written in the form $y = k(f - g) + g$ where k is an arbitrary constant and f, g are its particular solutions. BU(H) 2010, CU(H) -2009

10. Solve the problem $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$.

11. Solve

$$(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

12. Solve the differential equation $x\frac{dy}{dx} + y = y^2 \log x$

Answer any two from the Group -B: $2 \times 5 = 10$

1. Let M, N be two real-value functions which have continuous first partial derivatives on some rectangle

$$R: \ |x - x_0| \leq a, \ |y - y_0| \leq b, \quad (a, b > 0).$$

Then the necessary and sufficient conditions for the ordinary differential equation $M(x, y)dx + N(x, y)dy = 0$ to be exact in R is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ in } R.$$

2. Reduce the differential equation $(px^2 + y^2)(px + y) = (p+1)^2$ to Clairaut's form by the substitutions $u = xy, v = x + y$ and then obtain the complete primitive. **C.H.-92; V.H-00.**

3. **Prove that if $Mx + Ny \neq 0$ and the equation $Mdx + Ndy = 0$ be homogeneous differential equation where M, N have continuous first partial derivatives on some rectangle R, then $\frac{1}{Mx+Ny}$ in R is an integrating factor of the said equation.** **V.U(H) : 2016**

4. **Show that the general solution of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ can be written in the form $y = \frac{Q}{P} - e^{-\int Pdx}\left[e^{\int Pdx} d\left(\frac{Q}{P}\right) + c\right]$ where c is an arbitrary constant.** **V.U(H) : 2017**

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC.(H), 2nd Semester Internal Assessment-2021

Paper: C3T **Full Marks: 10** **Time: 2 Hour**

Real Analysis

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*Answer any "**Two**" questions* *2×5=10*

1. Let $S = \left\{(-1)^n \left(1 + \frac{1}{n}\right) : n \in N)\right\}$ 5
   (i) Show that -1 and 1 are limit points of S.
   (ii) Find the derived set of S.

2. Define open set with example. Prove that the intersection of a finite number of open sets in ***R*** is an open set. 5

3. Define bounded sequence. Prove that a convergent sequence is bounded. Is the converse true? Justify your answer. 5

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WhatsApp number: 9382292498

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC.(H), 2nd Semester Internal Assessment-2021

Paper: C3T **Full Marks: 10** **Time: 2 Hour**

Real Analysis

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*Answer any "**Two**" questions* *2×5=10*

1. Examine the set $\boldsymbol{S}$ is closed or open in $\boldsymbol{R}$. 5

   (i) $\boldsymbol{S} = \left\{x \in \boldsymbol{R}: \sin\frac{1}{x} = 0\right\}$

   (ii) $\boldsymbol{S} = \left\{\frac{1}{m} + \frac{1}{n}: m, n \in \boldsymbol{N}\right\}$

   (iii) $\boldsymbol{S} = \{x \in \boldsymbol{R}: 2x^2 - 5x + 2 < 0\}$

2. Let $\boldsymbol{S}$ be a non-empty subset of $\boldsymbol{R}$, bounded below and $\boldsymbol{T}= \{-x: x \in: S\}$. Prove that the set $\boldsymbol{T}$ is bounded above and $sup\ \boldsymbol{T} = -\inf \boldsymbol{S}$. 5

3. State and prove Cauchy's general principle of convergence. Find $\overline{lim} u_n$ where $u_n = (-1)^n(1 + \frac{1}{n})$. 4+1

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Mugberia Gangadhar Mahavidyalaya
Department of Mathematics (UG & PG),
B.SC.(H), 2nd Semester Internal Assessment-2022

Paper: C3T **Full Marks: 10** **Time: 1 Hour**

Real Analysis

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*Answer any "**Two**" questions* *2×5=10*

1. Define limit point and closed set. Let $\boldsymbol{S}$ be subset of $\boldsymbol{R}$. Prove that $\bar{\boldsymbol{S}} = \boldsymbol{S} \cup \boldsymbol{S}'$, $\boldsymbol{S}'$ being the derived set of $\boldsymbol{S}$. 5

2. Prove that the intersection of a finite number of open sets in $\boldsymbol{R}$ is an open set. Also examine the intersection of an infinite number of open sets in $\boldsymbol{R}$ is open or not? 5

3. Define bounded sequence. Prove that a convergent sequence is bounded. Is the converse true? Justify your answer. 5

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC.(H), 2nd Semester Internal Assessment-2022

Paper: C3T **Full Marks: 10** **Time: 1 Hour**

Real Analysis

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*Answer any "**Two**" questions* *2×5=10*

1. Define limit point and closed set. 5
Let $\boldsymbol{S}$ be subset of $\boldsymbol{R}$. Prove that $\bar{\mathbf{S}} = \mathbf{S} \cup \mathbf{S}'$, $\boldsymbol{S}'$ being the derived set of $\boldsymbol{S}$.

2. Prove that the intersection of a finite number of open sets in $\boldsymbol{R}$ is an open set. Also examine the intersection of an infinite number of open sets in $\boldsymbol{R}$ is open or not? 5

3. Define bounded sequence. Prove that a convergent sequence is bounded. Is the converse true? Justify your answer. 5

B.Sc 2nd Continuous Internal Assessment Examination 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Ordinary Differential Equations and Special Functions)

Paper MTM – CT4

FULL MARKS : 10 :: Time : 30 minutes

Answer any two questions **5*2=10**

1. Let W(f, g) be the wronskian of two linearly independent solutions f and g of the equation $\ddot{W} + P(z)\dot{W} + Q(z)W = 0$ **. Then find the value of product of W(f, g)P(z).**

2. **S**olve : $(1+3x)^2\frac{d^2y}{dx^2} - 6(1+3x)\frac{dy}{dx} + 6y = 8(1+3x)^2$, $-\frac{1}{3} < x < \infty$

3. Find the power series solution of the equation $4x^2y''(x) + 2xy'(x) - (x+4)y = 0$ in power of x.

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics

Differential Equations

Mathematics (Hons.): Part-I: Full Marks 90

Any twenty five from Group -A: $2 \times 25 = 50$

1. The type of the following differential equation $y'' + \sin(x+y) = \sin x$ is
   (a)linear,homogeneous (b)nonlinear,homogeneous
   (c)linear,nonhomogeneous (d)nonlinear, nonhomogeneous **Gate(MA): 2001**

2. If $y = \ln(\sin(x+a)) + b$ where a and b are constants, is the primitive, then the corresponding lowest order differential equation is
   (a) $y'' = -(1+(y')^2)$ (b) $y'' = 1+(y')^2$ (c) $y'' = -(2+(y')^2)$
   (d) $y'' = -(3+(y')^2)$ [JAM CA-2005]

3. The degree of $\frac{d^2y}{dx^2} = \log(y + \frac{dy}{dx})$ is
   (a) 1 (b) 0 (c) Does not exist (d) 2

4. Solution of the differential equation
   $xy' + \sin 2y = x^3 \sin^2 y$ is
   (a) $\cot y = -x^3 + c\,x^2$ (b) $2\cot y = -x^3 + 3c\,x^2$ (c) $\tan y = -x^3 + c\,x^2$
   (d) $2\tan y = -x^4 + 2c\,x^2$ [JAM CA-2005]

5. The Wronskian of the function $f_1(x) = x^2$ and $f_2(x) = x|x|$ is zero for
   (a) all x (b) $x > 0$ (c) $x < 0$ (d) $x = 0$ [JAM CA-2005]

6. The solution of the differential equation
   $y'' + 4y = 0$ subject to $y(0) = 1$, $y'(0) = 2$ is
   (a) $\sin 2x + 2\cos 2x$ (b) $\sin 2x - \cos 2x$ (c) $\sin 2x + \cos 2x$ (d) $\sin 2x + 2x$ [JAM CA-2005]

7. General solution of the differential equation
   $xdy = (y + x\,e^{-\frac{y}{x}})dx$ is given by
   (a) $e^{-\frac{y}{x}} = ln\,x + c$ (b) $e^{\frac{y}{x}} = ln\,x + c$ (c) $e^{-\frac{y}{x}} + ln\,x = c$ (d) $e^{-\frac{y}{x}} = x + c$ [JAM CA-2005]

8. The initial value problem

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0,\ y(0) = 1\ (\frac{dy}{dx})_{x=0} = 0$$

has **GATE(MA)-06**

A) a unique solution
B) no solution
C) infinitely many solution
D) two linearly independent solutions.

9. A particular solution of the differential equation $(D^4 + 2D^2 - 3)y = e^x$ is
(a) $(x+1)e^x$ (b) $\frac{xe^x}{8}$ (c) xe^x (d) $\frac{xe^x}{4}$ [JAM CA-2005]

10. The boundary value problem $y'' + \lambda y = 0$ satisfying $y(-\pi) = y(\pi)$ and $y'(-\pi) = y'(\pi)$ to each eigenvalue λ, there corresponds
(a) only one eigenfunction (b) two eigenfunctions
(c) two linearly independent eigenfunctions **NET(MS): (June)2011** (d) two orthogonal eigenfunctions

11. For the Sturm Liouville problems

$$(1+x^2)y^{''} + 2xy^{'} + \lambda x^2 y = 0$$

with $y^{'}(1) = 0$ and $y^{'}(10) = 0$ the eigenvalues, λ, satisfy **GATE(MA)-03**

A) $\lambda \geq 0$ B) $\lambda < 0$ C) $\lambda \neq 0$ D) $\lambda \leq 0$

12. The differential equation $ydx - (3y - 2x)dy = 0$
(a) exact and homogeneous but not linear (b) linear and homogeneous but not exact
(c) exact and linear but not homogeneous (d) exact, homogeneous and linear [JAM CA-2006]

13. The orthogonal trajectories of the curves
$y^2 = 3x^3 + x + c$ are
(a) $2\tan^{-1} 3x + 3ln|y| = k$ (b)$3\tan^{-1} 3x + 2ln|y| = k$
(c) $3\tan^{-1} 3x - 3ln|y| = k$ (d) $2\tan^{-1} 3x - 3ln|y| = k$ [JAM CA-2006]

14. The general solution of the differential equation $(6x^2 - e^{-y^2})dx + 2xye^{-y^2}dy = 0$ is
(a) $x^2(2x - e^{-y^2}) = c$ (b)$x^2(2x + e^{-y^2}) = c$
(c) $x(2x + e^{-y^2}) = c$ (d) $x(2x^2 - e^{-y^2}) = c$ [JAM CA-2006]

15. The orthogonal trajectories of the family of the curves $(x-1)^2 + y_2^2 ax = 0$ are the solution of the differential equation
(a) $x^2 - y^2 - 1 + 2xy\frac{dy}{dx} = 0$ (b)$x^2 - y^2 - 1 - 2xy\frac{dy}{dx} = 0$
(c) $x^2 - y^2 - 1 + 3xy\frac{dy}{dx} = 0$ (d) $x^2 + y^2 - 1 - 2xy\frac{dy}{dx} = 0$ [JAM CA-2008]

16. Let $W(y_1(x),\, y_2(x))$ is the Wronskian form for the solutions $y_1(x)$ and $y_2(x)$ of the differential equation $y'' + a_1 y' + a_2 y = 0$. If $W \neq 0$ for some $x = x_0$ in $[a,\ b]$ then
(a) it vanishes for any $x \in [a,\ b]$ (b) it does not vanishes for any $x \in [a,\ b]$
(c) it vanishes for only at $x = a$ (d) None [JAM CA-2009]

17. If general solution of the differential equation $y'' - m^2 y = 0$ is
(a)$c_1 \sinh mx + c_2 \cosh mx$ (b)$c_1 \sinh mx + c_2 \cos 2mx$
(c) $c_1 \sinh 2mx + c_2 \cosh mx$ (d) $c_1 \sinh mx + c_2 \coth mx$ [JAM CA-2009]

18. The general solution of the differential equation $y' = 2^{x-y}$ is
(a)$2^{-x} + 2^{-y} = c$ (b)$2^{-x} - 2^{-y} = c$
(c)$2^x + 2^y = c$ (d)$2^x - 2^y = c$
[JAM CA-2009]

19. The solution of the differential equation $\frac{d^2y}{dx^2} - y = e^x$ satisfying $y(0) = 0$ and $\frac{dy}{dx}(0) = \frac{3}{2}$ is
(a)$y(x) = \sinh x + \frac{x}{2}e^x$ (b)$y(x) = \sinh x - \frac{x}{2}e^x$
(c)$y(x) = \cosh x + \frac{x}{2}e^x$ (d)$y(x) = x\cosh x + \frac{x}{2}e^x$
[JAM CA-2010]

20. The boundary value problem $y'' + \lambda y = 0$ satisfying $y(-\pi) = y(\pi)$ and $y'(-\pi) = y'(\pi)$ to each eigenvalue λ, there corresponds
(a) only one eigenfunction (b) two eigenfunctions
(c) two linearly independent eigenfunctions **NET(MS): (June)2011** (d) two orthogonal eigenfunctions

21. For the Sturm Liouville problems

$$(1+x^2)y^{''} + 2xy^{'} + \lambda x^2 y = 0$$

with $y^{'}(1) = 0$ and $y^{'}(10) = 0$ the eigenvalues, λ, satisfy **GATE(MA)-03**

A) $\lambda \geq 0$ B) $\lambda < 0$ C) $\lambda \neq 0$ D) $\lambda \leq 0$

22. If $y = x\cos x$ is a solution of an n^{th} order linear differential equation $\frac{d^n y}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \cdots + a_{n-1}\frac{dy}{dx} + a_n y = 0$ with real constant coefficients, then the least possible value of n is
(a) 1 (b) 2 (c) 3 (d) 4 [JAM CA-2011]

23. The general solution of the differential equation $y'' = (y')^2$ is
(a)$x = c_1e^{-y} + c_2$ (b)$x = c_1e^{y} + c_2$
(c)$x = c_1e^{-y} + c_2y$ (d)$y = c_1e^{x} + c_2$
[JAM CA-2011]

24. The particular integral of the differential equation $y'' - 16y = 4\sinh^2 2x$ is
(a)$\frac{1}{8}(xe^{4x} - xe^{-4x} + 1)$ (b)$\frac{1}{8}(xe^{4x} + xe^{-4x} + 1)$
(c)$\frac{1}{4}(xe^{4x} - xe^{-4x} + 1)$ (d)$\frac{1}{8}(xe^{4x} - xe^{-4x} + 3)$ [JAM CA-2011]

25. Consider the differential equation
$\frac{dy}{dx} = ay - by^3$, where $a, b > 0$ and $y(0) = y_0$ As $x \to \infty$,the solution $y(x)$ tends to
(a)0 (b)$\frac{a}{b}$ (c)$\frac{b}{a}$ (d)y_0 [JAM MA-2009]

26. Consider the differential equation
$(x + y + 1)dx + (2x + 2y + 1)dy = 0$. Which of the following statements is true?
(a)The differential equation is linear (a)The differential equation is exact (c)e^{x+y} is an integrating factor of the differential equation (d)A suitable substitution transforms the differentiable equation to the variables separable form.
[JAM MA-2010]

27. The solution of the differential equation
$y'' + 4y = 0$ subject to $y(0) = 1$, $y'(0) = 2$ is
(a) $\sin 2x + 2\cos 2x$ (b) $\sin 2x - \cos 2x$ (c) $\sin 2x + \cos 2x$ (d) $\sin 2x + 2x$ [JAM CA-2005]

28. The solution of the boundary value problem
$y'' + y = cosecx,\ 0 < x < \frac{\pi}{2}$, $y(0) = 0$, $y(\frac{\pi}{2}) = 0$ is **NET(MS): (June)2012**
(a) convex (b) concave (c) negative (d) positive

29. Let V be the set of all bounded solution of the ODE
$u''(t) - 4u'(t) + 3u(t) = 0$, $t \in \Re$, Then V **NET(MS): (June)2012**
(a) ia a real vector space of dimension 2
(b) is a real vector space of dimension 1

(c) contains only the trivial solution $u = 0$
(d) contains exactly two solution

30. The set of all eigenvalues of

$$y^{''} + \lambda y = 0,\ y^{'}(0) = 0,\ y^{'}(\frac{\pi}{2}) = 0$$

is **GATE(MA)-04**

A) $\lambda = 2n,\ n = 1, 2, 3, \cdots$ B) $\lambda = 4n^2,\ n = 1, 2, 3, \cdots$ C) $\lambda = n,\ n = 0, 1, 2, 3, \cdots$
D) $\lambda = 4n^2,\ n = 0, 1, 2, 3, \cdots$

31. If $y_1(x)$ and $y_2(x)$ form a fundamental set of solutions of $y'' + p(x)y' + q(x)y = 0,\ a \leq x \leq b$,, where $p(x)$ and $q(x)$ are real-valued continuous function on an interval $[a,\ b]$. If x_0 and x_1 with $x_0 < x_1$ are consecutive zeros of $y_1(x)$ in (a, b), then
(a) $y_1(x) = (x - x_0)q_0(x)$ where $q_0(x)$ is continuous on $[a, b]$ with $q_0(x_0) \neq 0$,
(b) $y_1(x) = (x - x_0)^2 p_0(x)$ where $p_0(x)$ is continuous on $[a, b]$ with $p_0(x_0) \neq 0$,
(c) $y_2(x)$ has no zeros in (x_0, x_1)
(d) $y_2(x) = 0$ but $y_2'(x_0) \neq 0$ **[NET(MS)(Dec.)2011]**

32. Consider the equation of an ideal planer pendulum $\frac{d^2x}{dt^2} = -\sin x$ where x denotes the angle of displacement. For sufficiently small angles of displacement, the solution is given by (where A and B) are arbitrary constants **NET(MS): (June)2013**
(a) $x(t) = A\cosh t + B\sinh t$ (b) $x(t) = A + Bt$
(c) $x(t) = Ae^t + Be^{2t}$ (d) $x(t) = A\cos t + B\sin t$

Any ten from Group -B: $4 \times 10 = 40$

1. show that the differential equation of all parabolas with foci at the origin and axis along x-axis is given by
$y(\frac{dy}{dx})^2 + 2x\frac{dy}{dx} - y = 0$ [V.U.2002]

2. Solve : $(xy \sin\ xy + \cos\ xy)ydx + (xy \sin\ xy - \cos\ xy)xdy = 0$ [V.U.2002]

3. Reduce the equation
$x^2(\frac{dy}{dx})^2 + y(2x + y)\frac{dy}{dx} + y^2 = 0$ to Clairaut's form by the substitution $y = u, yx = v$
[V.U.2002]

4. Solve:$x^3\frac{d^3y}{dx^3} + 2x^2\frac{d^2y}{dx^2} - 2y = 10(x + \frac{1}{x})$ [V.U.2002]

5. Solve: $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = 3x^2e^{2x} \sin\ 2x$ [V.U.2002]

6. Solve: $\frac{dx}{dt} + 5x + y = e^t$
$\frac{dy}{dt} + 3y - x = e^2 t$ [V.U.2002]

7. Find the eigen values and eigen functions of the boundary value problem
$\frac{d^2y}{dx^2} + \lambda\, y = 0$ with $y(0) = 0$and $y(2\pi) = 0$. [V.U.2002]

8. Reducing the differential equation
$x^2p^2 + px(2x + y) + y^2 = 0$ to Clairaut's by the substitution $y = u, xy = v$ solve it and proved that $y + 4x = 0$ is a singular solution.

9. Solve the following differential equation by the method of variation of parameter
$\frac{d^2y}{dx^2} + y = \sec^3 x \tan x$

10. Show that the family confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1$ is self orthogonal.

11. Solve the differential equation
$x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = -4x^5$ given that $y = e^{x^2}$ is a part of the complementary function, $x > 0$.

12. Find the eigen values and eigen functions for differential equations $\frac{d^2y}{dx^2} + \lambda\, y = 0$ which satisfies the conditions $y(0) = 0$ and $y(\pi) = 0$.Examine whether it is a boundary value problem or initial value problem.

13. Deduce the necessary and sufficient condition for the ordinary differential equations $P(x, y)dx + Q(x, y)dy = 0$ to be exact.

14. Show that the equation of the curves, that cut a system of concentric circles $x^2+y^2 = a^2$ at an angle $\frac{\pi}{4}$,is $x^2 + y^2 = ae^{-2\tan^{-1}(\frac{y}{x})}$, where a being constant.

15. Reduced the differential equation $x^2p^2 + py(2x + y) + y^2 = 0$ to Clairaut's form by the substitutions $y = u,\ xy = v$, solve it for singular solution and extraneous loci, if any.

16. Let r_1, r_2 be the roots of the indicial polynomial for the equation
$$y'' + ay' + by = 0,$$
where a, b are constants.
(a) If $r_1 \neq r_2$, then show that two independents solutions are $e^{r_1x},\ e^{r_2x}$ on $[a, b]$
(b) If $r_1 = r_2$, then also show that the two independents solutions are given by
$$e^{r_1x},\ xe^{r_1x}.$$

Mugberia Gangadhar Mahavidyalaya
Department of Mathematics
Differential Equations
Mathematics (Hons.): Sem-I: Full Marks 31

Any seven from Group -A: $3 \times 7 = 21$

1. Show that the differential equation of all parabolas with foci at the origin and axis along x-axis is given by
$y(\frac{dy}{dx})^2 + 2x\frac{dy}{dx} - y = 0$ [V.U.2002]

2. Solve : $(xy \sin xy + \cos xy)ydx + (xy \sin xy - \cos xy)xdy = 0$ [V.U.2002]

3. Reduced the differential equation $x^2p^2 + py(2x + y) + y^2 = 0$ to Clairaut's form by the substitutions $y = u,\ xy = v$, solve it for singular solution and extraneous loci, if any.

4. Show that the substitution $x = e^u$ transforms the equation
$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = \cos x$ into $\frac{d^2y}{du^2} + 3\frac{dy}{du} + 2y = \cos x$. JAM(MA)–2010

5. Prove that the differential equation of the circles through the intersection of the circle $x^2 + y^2 = 1$ and the line $x - y = 0$ is
$(x^2 - 2xy - y^2 + 1)dx + (x^2 + 2xy - y^2 - 1)dy = 0$ **V.U(Hons.)-2017**

6. Explain the terms: general solution, a particular solution, a singular solution as applied to an ordinary differential equation.

7. The equation
$$\frac{x^2}{a^2 + \lambda} + \frac{y^2}{b^2 + \lambda} = 1, \qquad (1)$$
WBSSC 2001
(where a and b are fixed constants and λ is an arbitrary parameter which can assume all real values) represents a family of confocal conics. To obtain the differential equation of this family.

8. If $\frac{1}{M-N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x + y)$, then the differential equation $Mdx + Ndy = 0$ has an integrating factor of the form $e^{-\int f(x+y)d(x+y)}$.

9. Show that the general solution of the differential equation $\frac{dy}{dx} + P(x)y = Q(x)$ can be written in the form $y = k(f - g) + g$ where k is an arbitrary constant and f, g are its particular solutions. BU(H) 2010, CU(H) -2009

10. Solve the problem $\frac{dy}{dx} = \frac{y-x+1}{y+x+5}$.

11. Solve

$$(1+x)\frac{dy}{dx} - y = e^{3x}(x+1)^2$$

12. Solve the differential equation $x\frac{dy}{dx} + y = y^2 \log x$

All from Group -B: $2 \times 5 = 10$

1. Let M, N be two real-value functions which have continuous first partial derivatives on some rectangle

$$R: \ |x - x_0| \leq a, \ |y - y_0| \leq b, \quad (a,\, b > 0).$$

Then the necessary and sufficient conditions for the ordinary differential equation $M(x,y)dx + N(x,y)dy = 0$ to be exact in R is

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \ \text{ in } \ R.$$

2. The number of integrating factors of an equation $M(x,y)dx + N(x,y)dy = 0$ is infinite on R.

B.Sc 2nd Continuous Internal Assessment Examination 2021

Department of Mathematics, Mugberia Gangadhar Mahavidyalaya

(Ordinary Differential Equations and Special Functions)

Paper MTM – CT4

FULL MARKS : 10 :: Time : 30 minutes

Answer any two questions **5 +5=10**

1. Solve the following LPPs using the simplex method:

$$Maximize\ z = 10x_1 + x_2 + 2x_3$$

$$Subject\ to\ \begin{array}{c} x_1 + x_2 - 2x_3 \le 10, \\ 4x_1 + x_2 + x_3 \le 20, \\ x_1, x_2, x_3 \ge 0. \end{array}$$

2. Make a graphical representation of the set of constraints of the following LPP. Find the corner points of the feasible region. Then solve the problem graphically.

$$Minimize\ z = 4x_1 + 2x_2$$

$$Subject\ to\ \begin{array}{c} 3x_1 + x_2 \ge 27, \\ -x_1 - x_2 \le -21, \\ x_1 + 2x_2 \ge 30, \\ x_1, x_2 \ge 0. \end{array}$$

3. Solve the Legendre differential equation of the form

$$(1-z^2)\frac{d^2y}{dz^2} - 2z\frac{dy}{dz} + n(n+1)y = 0.$$

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC., 3rd Semester Internal Assessment 2022

Full Marks: 10 **Time: 1/2 Hour**

Paper: C7T-NUMERICAL METHODS

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*Answer **"any one"** of the following questions:*

1.a	Explain when relative error is a better indicator of the accuracy of a computation than the absolute error.	2
1.b	Compare bisection method and regula-falsi method.	2
1.c	Using LU decomposition method, solve the following system of equations	5
1.d	Test if the following system of equations is diagonally dominant:	1

1.c equations:

$$x_1 + x_2 + x_3 = 3$$
$$2x_1 - x_2 + 3x_3 = 16$$
$$3x_1 + x_2 - x_3 = -3$$

1.d equations:

$$10x + 15y + 3z = 14$$
$$-30x + y + 5z = 17$$
$$x + y + 4z = 3$$

2.a Describe Newton-Raphson method for computing a simple real root of an equation $f(x) = 0$. Give a geometrical interpretation of the method. What are the advantages and disadvantages of this method? 2+1+1

2.b Solve the following equations by Gauss-Seidal's method, correct up to four significant figures: 4

$$9x + 2y + 4z = 20$$
$$x + 10y + 4z = 6$$
$$2x - 4y + 10z = -15.$$

2.c Find the rate of convergence of secant methodfor computing a simple real root of an equation $f(x) = 0$. 2

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC., 3rd Semester Internal Assessment 2022

Full Marks: 10 **Time: 1 Hour**

Paper: C7T- NUMERICAL METHODS

---

*Answer "**any one**" of the following questions:*

1.1	Explain when relative error is a better indicator of the accuracy of a computation than the absolute error.	2
1.2	Compare bisection method and regula-falsi method.	2
1.3	Using LU decomposition method, solve the following system of equations $$x_1 + x_2 + x_3 = 3$$ $$2x_1 - x_2 + 3x_3 = 16$$ $$3x_1 + x_2 - x_3 = -3$$	5
1.4	Test if the following system of equations is diagonally dominant: $$10x + 15y + 3z = 14$$ $$-30x + y + 5z = 17$$ $$x + y + 4z = 3$$	1
2.1	Describe Newton-Raphson method for computing a simple real root of an equation $f(x) = 0$. Give a geometrical interpretation of the method. What are the advantages and disadvantages of this method?	2+1+1
2.2	Solve the following equations by Gauss-Seidal's method, correct up to four significant figures: $$9x + 2y + 4z = 20$$ $$x + 10y + 4z = 6$$ $$2x - 4y + 10z = -15.$$	4
2.3	Find the rate of convergence of secant method for computing a simple real root of an equation $f(x) = 0$.	2

Exam attendance link (must): https://forms.gle/SSBCtRkWTYXYRbq17

Submit your ANSWER SCAN PDF Copy using either Attendance link above or the following Email: manoranjande.math.rs@jadavpuruniversity.in

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG)

B.Sc. (Hons) 4th Semester

1st internal Assesment -2023

(Metric Space and Complex Analysis)

Paper : C8T

Full Marks : 10 :: Time : 1/2 hour

1. **Answer any two question** **5×2**

   a. Let a function $f:[a,b] \to \mathbb{R}$ be bounded on [a, b] and let f be continuous on [a, b] except for a finite number of points in [a, b]. Then f is integrable on [a, b].

   b. Define uniform convergent of a sequence. A sequence of function $\{f_n\}$ is defined by $f_n(x) = \frac{nx}{1+n^2x^2}$, $0 \leq x \leq 1$. Show that the sequence $\{f_n\}$ is not uniformly convergent on [0, 1].

   c. A function f is defined on [0, 1] by f(x)=$\sin x$, if x is rational and f(x)=x , if x is irrational (i) Evaluate upper and lower integral of f on [0, $\frac{\pi}{2}$] . (ii) Show that f is not integrable on [0, $\frac{\pi}{2}$].

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG)

B.Sc. (Hons) 4th Semester

1st internal Assesment -2023

(Metric Space and Complex Analysis)

Paper : C8T

Full Marks : 10 :: Time : 1/2 hour

1. **Answer any two question** **5×2**
   a. Let a function $f: [a, b] \to \mathbb{R}$ be bounded on [a, b] and let f be continuous on [a, b] except for a finite number of points in [a, b]. Then f is integrable on [a, b].
   b. Define uniform convergent of a sequence. A sequence of function $\{f_n\}$ is defined by $f_n(x) = \frac{nx}{1+n^2x^2}$, $0 \le x \le 1$. Show that the sequence $\{f_n\}$ is not uniformly convergent on [0, 1].
   c. A function f is defined on [0, 1] by f(x)=$\sin x$, if x is rational and f(x)=x , if x is irrational (i) Evaluate upper and lower integral of f on [0, $\frac{\pi}{2}$] . (ii) Show that f is not integrable on [0, $\frac{\pi}{2}$].

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC.(H), 5th Semester Internal Assessment-2021

Paper: DSE-1T **Full Marks: 10** **Time: 2 Hour**

Linear Programming

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*Answer any "**One**" question* *1×10=10*

1.1 Prove that the transportation problem always has a feasible solution. 2

1.2 Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix 4

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

1.3 Solve graphically or otherwise the game whose pay of matrix is 4

	B1	B2	B3	B4
A1	2	2	3	-1
A2	4	3	2	6

2.1 Find all the basic solution of the system 2

$$2x_1 + x_2 + 4x_3 = 11$$
$$3x_1 + x_2 + 5x_3 = 14$$

2.2 Find graphically the non-negative values of the variables x_1 and x_2 which satisfy the constraints $3x_1 + 5x_2 \leq 15$, $5x_1 + 2x_2 \leq 10$ and which maximize the linear form $z = 5x_1 + 3x_2$ 4

2.3 Solve by simplex method (Big M-method) 4

$$\begin{aligned} &Maximize &&5x_1 + 8x_2 \\ &Subject\ to &&3x_1 + 2x_2 \geq 3, \\ & &&x_1 + 4x_2 \geq 4, \\ & &&x_1 + x_2 \leq 5, \\ & &&x_1,\ x_2 \geq 0. \end{aligned}$$

Exam attendance link (must) : https://forms.gle/ZRztD1zuCxGbZwcz6

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Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC.(H), 5th Semester Internal Assessment-2021

Paper: DSE-1T **Full Marks: 10** **Time: 1 Hour**

Linear Programming

---

*Answer any "**One**" question of the followings:* *1×10=10*

1.1 What is unbalanced transportation problem (TP)? Prove that the balanced TP always has a feasible solution. 2+3

1.2 Find the optimal assignments to find the minimum cost for the assignment problem with the following cost matrix 5

	I	II	III	IV	V
A	6	5	8	11	16
B	1	13	16	1	10
C	16	11	8	8	8
D	9	14	12	10	16
E	10	13	11	8	16

2. Four products are produced in three machines and their profit margins are given by the table below: 5 + 2 + 3

	P1	P2	P3	P4	Capacity
M1	6	4	1	5	**14**
M2	8	9	2	7	**18**
M3	4	3	6	2	**7**
Requirements	**6**	**10**	**15**	**8**	

a) Find a suitable production plan of products in machines so that the capacities and requirements are met and the profit is maximized.

b) How can you identify an alternate solution? Can you find a similar situation in the problem?

c) While evaluating alternative solution, you may meet a case of degeneracy for testing the optimality criteria. Explain the step to avoid degeneracy in the problem.

3.1 Give the comparison between transportation and assignment problem. 2

3.2 What are the restrictions required to adopt in a Travelling Salesman Problem (TSP)? How can you solve a TSP? 2+1

3.3 Solve the following TSP 5

	A	B	C	D	E
A	--	24	17	11	19
B	24	--	18	16	11
C	17	18	--	15	9
D	11	16	15	--	21
E	19	11	9	21	--

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics :: Class Text(2019)

Partial Differential Equations

Mathematics (Hons.): SEM-V: CT11: Full Marks 40

Any five from Group -A: $2 \times 5 = 10$

1. Find the general solution of the PDE $uu_x + yu_y = x$.
2. Find the partial differential equation by eliminating the arbitrary constants a and b from $z = (x^2 + a)(y^2 + b)$.
3. Find the order and degree of the PDE $p \tan y + q \tan x = \sec^2 z$.
4. Let $u(x,t)$, $x \in \Re$, $t \geq 0$ be the solution of the initial value problem $u_{xx} = u_{tt}$, $u(x,0) = x$ and $u_t(x,0) = 1$. Then find the value of $u(2,2)$.
5. Let $a, b \in \Re$ be such that $a^2 + b^2 \neq 0$. Then verify that the Cauchy problem $au_x + bu_y = 1$, $x, y \in \Re$ with $u(x,y) = x$ on $ax + by = 1$ has a unique solution or not ?
6. The second order PDE $u_{yy} - yu_{xx} + x^3 u = 0$ is **NET(MS): (June)2012**
   (a) Elliptic for all $x \in \Re$, $y \in \Re$ (b) Parabolic for all $x \in \Re$, $y \in \Re$
   (c) Elliptic for all $x \in \Re$, $y < 0$ (d)Hyperbolic for all $x \in \Re$, $y < 0$.
7. Find characteristic curve of the following PDEs :
   (a) $yz\frac{\partial z}{\partial x} + xz\frac{\partial z}{\partial y} = xy$
   (b) $yz\frac{\partial z}{\partial x} + xz^2\frac{\partial z}{\partial y} = xy$.

Any six questions from Group -B: $5 \times 6 = 30$

1. Find the integral surface of the linear PDE $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0$, $z = 1$.
2. Find the equation of the integral surface of $x^2 p + y^2 q + z^2 = 0$ which passes through the hyperbola $xy = x + y$, $z = 1$
3. Find the equation of the integral surface satisfying $4yzp + q + 2y = 0$ and passing through the curve $y^2 + z^2 = 1$, $x + z = 2$ **IAS 1997**
4. Show that the equations $xp - yq = 0$, $z(xp + yq) = 2xy$ are compatible and solve them.
   Ans. $z^2 = 2xy + k$ where k is a constant.

5. Reduce the following PDEs to canonical form : $x^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} - x\frac{\partial z}{\partial x} + 3y\frac{\partial z}{\partial y} - \frac{8y}{x} = 0$.
6. Find the solution of the equation $2z = p^2 + q^2 + 2(p - x)(q - y)$ which passes through the x-axis. **IAS 2002**

7. Find a complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ **IAS 1991**
8. Find the characteristics of the equation $p^2 + q^2 = 2$ and determine the integral surface which passes through the straight line $x = 0,\ z = y$.
9. Using the method of separation of variables solve

$$4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u, \quad \text{where} \quad u(0, y) = 3e^{-y} - e^{-5y}$$

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG)

B.Sc. (Hons) 6th Semester

2nd internal Assesment -2023

(Metric Space and Complex Analysis)

Paper : C13T

Full Marks : 10 :: Time : 1/2 hour

Answer any one question. **10x1=10**

1. a) Let C denote the unit circle centered at the origin in $\mathbb{C}$. Then find the value of $\frac{1}{2\pi i}\int |1 + z + z^2|^2\, dz$, where the integral is taken anti-clockwise along C.

   b) Determine the nature of the singularity at z = 0 of the function $f(z) = (e^z+1)/(e^z - 1)$. Hence find the residue of $f(z)$ at z = 0.

   c)Let P[0,1] be the set of all polynomials defined on [0,1]. Show that
   $$d(P_1, P_2) = \sup_{0\le x\le 1} |P1(x) - P2(x)|$$
   is a metric on P[0,1]. Also show that this metric space is incomplete.

   [3+2+5]

2. a) Find the interval of convergence of the power series $\sum_{n=0}^{\infty} 3^{-n}(z-1)^{2n}$.

   b) Let C be the counter-clockwise oriented circle of radius $\frac{1}{2}$ centered at *i*. Then find the value of $\oint \frac{dz}{z4-1}$ on C.

   c) State and prove Cantor's intersection theorem on a complete metric space. [2+3+5]

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG)

B.Sc. (Hons) 6th Semester 2023

1st internal Assesment

(Metric Space and Complex Analysis)

Paper : C13T

Full Marks : 10 :: Time : 1/2 hour

1. Answer any one question **4×1**

a) Show that every Cauchy sequence in a metric space is bounded, but the converse is not true.

b) Prove that a convergent sequence $\{x_n\}$ in (X,d) is a Cauchy sequence. Give an example to show that a Cauchy sequence need not be convergent in an arbitrary metric space.

2. Answer any one question **6×1**

a) i) State and prove Cauchy's integral formulae.

ii) Prove that $\nabla^2 \equiv 4\frac{\partial z}{\partial z \partial \bar{z}}$ [4+2]

b) i) Evaluate on C: $\oint \frac{e^{2z}}{(z+1)4} dz$, where C is the circle |z|=3.

ii) State Laurent's theorem and hence define the removable singularity, pole and isolated essential singularity. [3+3]

Mugberia gangadhar mahavidyalaya

Department of mathematics

B.Sc 6th Semester examination, 2022

Paper-C14T

Internal assessment

FULL MARKS : 10

Answer any one question $\mathbf{10 \times 1 = 10}$

1. (a) If $f(x)$ is a polynomial in $F(x)$ of degree 2 or 3 , then show that$f(x)$ is reducible over the field F iff it has a zero in F.

   (b) Define dual space of a vector space V.If V is a vector space of dimension n over a field F. Then the dimension of its dual space is also n.

2. (a) Consider the integral domain $\mathbb{Z}[\sqrt{-3}]$ then show that

   (i) 1 and -1 are the only units in this integral domain.

   (ii) $1+\sqrt{-3}$,2 are irreducible element in this integral domain.

   (iii) But none of $1+\sqrt{-3}$s and 2 is prime there.

   (b) Find the orthogonal complement of the subspace P , generated by the vectors (1,1,0) and (0,1,1) in $\mathbb{R}^3$.

B.Sc. 6th SEM 2nd Internal Examination, 2023

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

(Mathematical Modeling)

Paper DSE– 4

FULL MARKS: 10 : Time: 30 min.

Answer any two questions of the following: $5\times2 = 10$

1. a. Use the convolution theorem to evaluate $L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$.

   b. Apply the convolution theorem to prove that
   B (m, n) = $\int_0^1 u^{m-1}(1-u)^{n-1}\, du = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, m> 0 , n> 0 .

2. Solve (tD^2+(1-2t)D -2) y = 0 ,y(0) =1, y^1(0) = 2 , where D$\equiv \frac{d}{dx}$.

3. Evaluate L$\{\int_0^t \frac{\sin u}{u}\, du\}$ by the help of initial value theorem.

Department of Mathematics,
B.SC., 6th Semester-2020

Paper: DSE-4 **Full Marks: 45** **Time: 1&1/2 Hour**

UNIT-II: Mathematical Modelling

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1. Answer the following questions *5+5+5=15*

1.1. Describe Monte Carlo Algorithm to find the area under a curve. Using Monte Carlo simulation, write an algorithm to calculate that part of volume of an ellipsoid $\frac{x^2}{2} + \frac{y^2}{4} + \frac{z^2}{8} \leq 16$ that lies on the first octant $x \geq 0,\ y \geq 0,\ z \geq 0$

1.2. Write the proper equation of linear congruence method by which random number is generated. Using this method, find five random number between 50 and 100.

1.3. What are the disadvantages of middle square method to generate random numbers? Generate 5 random numbers using this method and initially taking $x_0 = 653217$.

---

2. Answer the following questions *5+10=15*

2.1. Two different products, P1 and P2, can be manufactured by one or both of two different machines, M1 and M2. The unit processing time of either product on either machine is the same. The daily capacity of machine M1 is **200** units (of either P1 or P2, or a mix of both), and the daily capacity of machine M2 is **250** units. The shop supervisor wants to balance the production schedule of the two machines such that the total number of units produced on one machine is within **5** units of the number produced on the other. The profit per unit of P1 is **$10** and that of P2 is **$15**. Set up the problem as an LP in equation form.

2.2. Solve the following LPPs using simplex method:

$$Maximize\ z = 5x_1 - 2x_2 + 3x_3$$

$$Subject\ to\quad \begin{array}{l} 2x_1 + 2x_2 - x_3 \geq 2 \\ 3x_1 - 4x_2 \leq 3 \\ x_2 + 3x_3 \leq 5 \\ x_1, x_2, x_3 \geq 0. \end{array}$$

---

3. Answer the following questions *4+4+7=15*

3.1. Solve the following LPP by graphical method

$$Maximize\quad 60x + 50y$$

$$Subject\ to\quad \begin{array}{l} x + 2y \leq 1000 \\ 4x + 2y \leq 1600 \\ x, y \geq 0 \end{array}$$

Department of Mathematics,
B.SC., 6th Semester-2020

Paper: DSE-4 **Full Marks: 45** **Time: 1&1/2 Hour**

UNIT-II: Mathematical Modelling

3.2. Discuss the sensitivity of changes of the cost co-efficient in the objective function of a LPP associated with both basic and non-basic variables.

3.3. Find the optimal solution of the LPP:

$$\begin{array}{ll} \textit{Maximize} & z = 4x_1 + 5x_2 \\ & 3x_1 + 4x_2 \leq 14, \\ \textit{Subject to} & 4x_1 + 2x_2 \leq 8, \\ & 2x_1 + x_2 \leq 6, \\ & x_1, x_2 \geq 0. \end{array}$$

Show that the optimality of the solution is not violated if the right hand side of the first constraint varies between 6 and 16. Show further that the range of c_2 is $\left(\frac{5}{2}, \frac{20}{3}\right)$ in order that the optimal solution obtained remains optimal.

Department of Mathematics,
B.SC., 6th Semester-2020

Paper: DSE-4 **Full Marks: 60** **Time: 2 Hours**

UNIT-II: Mathematical Modelling

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1. **a)** What is a pseudorandom number? Write the application areas of it. Use the middle-square method to generate five random numbers using $x_0 = 3043$. **5+5+5**

b) Use the linear congruence method to generate 20 random numbers using a=5, b=3, and c=16. Comment about the results of each sequence. Was there cycling? If so, when did it occur?

c) Using Monte Carlo simulation, write an algorithm to find the area trapped between the two curves $y = x^2$ and $y = 6 - x$ and the x- and y-axes.

2. **a)** Solve the following LPPs using the simplex method: **10+5**

$$Maximize\ z = 10x_1 + x_2 + 2x_3$$

$$Subject\ to\ \begin{array}{c} x_1 + x_2 - 2x_3 \leq 10, \\ 4x_1 + x_2 + x_3 \leq 20, \\ x_1, x_2, x_3 \geq 0. \end{array}$$

b) Make a graphical representation of the set of constraints of the following LPP. Find the corner points of the feasible region. Then solve the problem graphically.

$$Minimize\ z = 4x_1 + 2x_2$$

$$Subject\ to\ \begin{array}{c} 3x_1 + x_2 \geq 27, \\ -x_1 - x_2 \leq -21, \\ x_1 + 2x_2 \geq 30, \\ x_1, x_2 \geq 0. \end{array}$$

3. Firestone, headquartered in Akron, Ohio, has a plant in Florence, South Carolina, that Manufactures two types of tires: **SUV 225** radials and **SUV 205** radials. Demand is high because of the recent recall of tires. Each batch of **100 SUV 225** radial tires requires **100 gals** of synthetic plastic and **5 lb** of rubber. Each batch of **100 SUV 205** radial tires requires **60 gals** of synthetic plastic and **2.5 lb** of rubber. Labor costs are **$1** per tire for each type of tire. The manufacturer has weekly quantities available of **660 gals** of synthetic plastic, **$750** in the capital, and **300 lb** of rubber. The company estimates a profit of **$3** on each **SUV 225** radial and **$2** on each **SUV 205** radial. **7+4+4**

a) How many of each type of tire should the company manufacture in order to maximize its profits?

b) Assume now that the manufacturer has the opportunity to sign a nice contract with a tire outlet store to deliver at least **500 SUV 225** radial tires and at least **300 SUV 205** radial tires. Should the manufacturer sign the contract? Support your recommendation.

c) If the manufacturer can obtain an additional **1000 gal** of synthetic plastic for a total cost of **$50**, should he choose this option? Support your recommendation.

Department of Mathematics,

B.SC., 6th Semester-2020

Paper: DSE-4 **Full Marks: 60** **Time: 2 Hours**

UNIT-II: Mathematical Modelling

4. **a)** Discuss the sensitivity of variations in the requirement vector of a standard LPP

b) Find the optimal solution of the LPP **5 +10**

$$\begin{aligned} &\text{Maximize} \quad && z = 4x_1 + 3x_2 \\ &\text{Subject to} && x_1 + x_2 \le 5, \\ & && 3x_1 + x_2 \le 7, \\ & && x_1 + 2x_2 \le 10, \\ & && x_1, x_2 \ge 0. \end{aligned}$$

Show how to find the optimal solution to the problem if

i) The first component of the original requirement vector be increased by one unit, and the third component be decreased by one unit;

ii) Two units decrease from the second component of the original requirement vector.

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics,
B.SC., 6th Semester Internal Assessment-2020

Paper: DSE-4 **Full Marks: 15** **Time: 1 Hour**

UNIT-II: Mathematical Modelling

Answer the following questions. *3×5=15*

1. Describe Monte Carlo Algorithm to find the area under a curve. Using Monte Carlo simulation, write an algorithm to calculate that part of volume of an ellipsoid $\frac{x^2}{2}+\frac{y^2}{4}+\frac{z^2}{8} \le 16$ that lies on the first octant $x \ge 0,\ y \ge 0,$ $z \ge 0$.

2. Write the proper equation of linear congruence method by which random number is generated. Using this method, find five random number between 50 and 100.

3. What are the disadvantages of middle square method to generate random numbers? Generate 5 random numbers using this method and initially taking $x_0 = 653217$.

Department of Mathematics,
Question B.SC. 6th Semester-2020

Paper: DSE-4 **Full Marks: 15** **Time: 1&1/2 Hour**

UNIT-II: Mathematical Modelling

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1. *Answer the following questions* *$5+(5+2\frac{1}{2}+2\frac{1}{2})=15$*

1.1. Use Monte Carlo simulation to approximate the area under the curve $f(x) = \sqrt{x}$ over the interval $\frac{1}{2} \leq x \leq \frac{3}{2}$.

1.2. A farmer has **30** acres on which to grow tomatoes and corn. Each **100** bushels of tomatoes require **1000** gallons of water and **5** acres of land. Each **100** bushels of corn require **6000** gallons of water and **2.5** acres of land. Labour costs are **$1** per bushel for both corn and tomatoes. The farmer has available **30,000** gallons of water and **$750** in capital. He knows that he cannot sell more than **500** bushels of tomatoes or **475** bushels of corn. He estimates a profit of **$2** on each bushel of tomatoes and **$3** on each bushel of corn.

a) How many bushels of each should he raise to maximize profits?

b) Next, assume that the farmer has the opportunity to sign a nice contract with a grocery store to grow and deliver at least **300** bushels of tomatoes and at least **500** bushels of corn. Should the farmer sign the contract? Support your recommendation.

c) Now assume that the farmer can obtain an additional **10,000** gallons of water for a total cost of **$50**. Should he obtain the additional water? Support your recommendation.

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics (UG & PG),

B.SC., 6th Semester 1st Internal Assessment-2023

Paper: DSE-4 **Full Marks: 10** **Time: 1/2 Hour**

UNIT-II: Mathematical Modelling

Answer any two of the following questions: *5×2=10*

1. Describe Monte Carlo Algorithm to find the area under a curve. Using Monte Carlo simulation, write an algorithm to calculate that part of volume of an ellipsoid $\frac{x^2}{2}+\frac{y^2}{4}+\frac{z^2}{8}\leq 16$ that lies on the first octant $x\geq 0,\ y\geq 0,$ $z\geq 0$.

2. Write the proper equation of linear congruence method by which random number is generated. Using this method, find five random number between 50 and 100.

3. What are the disadvantages of middle square method to generate random numbers? Generate 5 random numbers using this method and initially taking $x_0 = 653217$.

Mugberia Gangadhar Mahavidyalaya

Department of Mathematics

B.Sc 6th Semester examination, 2022

(Mathematical Modeling)

Paper MTM – DSE-4

2nd Internal Assessment Examination

FULL MARKS : 10 :: Time : 30 minutes

Answer any two questions from the following **10**

1. Show that when n is a positive integer, $J_n(x)$ is the coefficient of z^n in the expansion of $\exp\left(\frac{x(z-\frac{1}{z})}{2}\right)$. **5**

2. Prove that for the Bessel's function $2J_n^{'}(x) = J_{n-1}(x) - J_{n+1}(x)$ **5**

3. Establish the Bessel integral equation. 5

4. Solve the following LPPs using the simplex method:

$$Maximize\ z = 10x_1 + x_2 + 2x_3$$

$$Subject\ to\ \begin{array}{c} x_1 + x_2 - 2x_3 \leq 10, \\ 4x_1 + x_2 + x_3 \leq 20, \\ x_1, x_2, x_3 \geq 0. \end{array}$$

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